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Real symplectic formulation of local special geometry

Sergio Ferrara[♭] and Óscar Maciá[♯]

[♭] *Physics Department, Theory Unit, CERN,*
CH-1211 Geneva 23, Switzerland
e-mail: Sergio.Ferrara@cern.ch

[♭] *INFN, Laboratori Nazionali di Frascati,*
Via Enrico Fermi 40, I-00044 Frascati, Italy

[♯] *Departament de Física Teòrica, Universitat de València and IFIC*
C/Dr. Moliner, 50, E-46100 Burjassot (València), Spain.
e-mail: Oscar.Macia@ific.uv.es

Abstract

We consider a formulation of local special geometry in terms of Darboux special coordinates $P^I = (p^i, q_i)$, $I = 1, ..., 2n$. A general formula for the metric is obtained which is manifestly $\mathbf{Sp}(2n, \mathbb{R})$ covariant. Unlike the rigid case the metric is not given by the Hessian of the real function $S(P)$ which is the Legendre transform of the imaginary part of the holomorphic prepotential. Rather it is given by an expression that contains S, its Hessian and the conjugate momenta $S_I = \frac{\partial S}{\partial P^I}$. Only in the one-dimensional case ($n = 1$) is the real (two-dimensional) metric proportional to the Hessian with an appropriate conformal factor.

1 Introduction

Local special geometry [1], [2], [3], [4], [5], is the underlying geometry of $N{=}2$ supergravity in four dimensions coupled to vector multiplets. It also emerges in any compactification, obtained from a higher-dimensional theory, when eight supersymmetries are preserved in four dimensions or, in a more general setting, when the lower-dimensional theory can be viewed as a "deformation" of a $N = 2$ locally supersymmetric Lagrangian. These theories include Calabi–Yau compactifications from *type II* supergravity [3], [4] as well as compactifications on more general manifolds with $G = SU(3)$ structures [6], [7], [8], [9], [10]. Special geometry also plays an important role in the physics of black holes, attractor equations [11], [12], [13], [14], which control the horizon geometry, and more recently it has been used also to connect the black hole entropy to topological partition functions in superstring theory [15], [16], [17].

In this framework it is useful to reconsider a formulation of special geometry in real [18], [19], [20], rather than complex variables. Real variables, will be denoted as Darboux coordinates $P^I = (p^i, q_i)$, $i = 1, ..., n$ where n is the complex dimension of the (local) special manifold. One of the reasons to adopt these coordinates is that the attractor equations can be written as real equations [11], [12], [13], [14], [21], [22], [23], which determine the real charge vector $Q = (m^\Lambda, e_\Lambda)$, $(\Lambda = 0, 1, ..., n)$ in terms of the holomorphic sections $V = (X^\Lambda, F_\Lambda = \partial_\Lambda F)$ of special geometry [1], [2], [3], [5]; A direct real formulation of the geometry may therefore have a simplifying role.

In this note we extend a previous investigation [24] on the real formulation of rigid special geometry to the local case. The main result is the derivation of a general formula for the metric in special Darboux coordinates and also comment on the main differences from the rigid case.

A very important role is also played here by a $2n \times 2n$ symplectic real symmetric matrix $M(\Re f_{ij}, \Im f_{ij})$ constructed in terms of the holomorphic matrix

$$f_{ij} = \frac{\partial^2 f(t)}{\partial t^i \partial t^j}$$

where $t^i = \frac{X^i}{X^0}$ are the "special coordinates" of the local special geometry and $f(t)$ is the holomorphic prepotential in special coordinates. As in the rigid case the M matrix (which satisfies $M\Omega M = \Omega$ where Ω is the invariant symplectic form of $\mathbf{Sp}(2n, \mathbb{R})$) is related to the Hessian

$$H_{IJ} = \frac{\partial^2 S}{\partial P^I \partial P^J}$$

of a certain Hamiltonian function $S(p, q)$ written in Darboux special coordinates but, unlike the rigid case, the general formula for the metric is given by

$$g_{IJ}(P) = -\frac{1}{2S} H_{IJ} + \frac{1}{4S^2} \left(S_I S_J + (H\Omega S)_I (H\Omega S)_J \right) \tag{1}$$

The first term is analogous to the term that is present in the rigid case. The second term, due to the Hodge-Kähler structure of local special geometry, has no rigid analogue and depends, other

2

than S and its Hessian, also on the conjugate momenta $S_I = \frac{\partial S}{\partial P^I}$, of the Darboux coordinates P^I. Note that $(H\Omega S)_I$ denotes the expression

$$(H\Omega S)_I = H_{IK}\Omega^{KL}S_L \quad , \quad \Omega^{KL} = -\Omega_{KL}$$

This paper is organized as follows. In section 2 we describe local special geometry in holomorphic and Darboux special coordinates and compute the Kähler metric in real symplectic coordinates. In section 3 we compute the (real symmetric) metric in the real symplectic Darboux coordinates, and we show the explicit $\mathbf{Sp}(2n, \mathbb{R})$ covariance of this metric and its differences from the rigid case. In section 4 some examples are given.

2 Local special geometry in Darboux coordinates

We consider local special geometry in special coordinates $t^\Lambda = \frac{X^\Lambda}{X^0} = (t^0 = 1, t^i)$. In these coordinates the holomorphic symplectic sections

$$V = (X^\Lambda, \partial_\Lambda F) \tag{2}$$

take the simple form

$$V = \left(1 \; , \; t^i \; ; \; f_0 = 2f - t^i f_i \; , \; f_i = \frac{\partial f}{\partial t^i}\right) \tag{3}$$

in terms of the prepotential $f(t) = (X^0)^{-2}F(X)$ where $F(X)$ is homogeneous of degree 2 [1]

$$X^\Lambda \partial_\Lambda F = 2F$$

From this, it follows that $f(t)$ is a fairly arbitrary holomorphic function fo t^i. Moreover t^i are "scalar" sections of the Hodge bundle since under a holomorphic gauge transformation

$$(X^\Lambda, F_\Lambda) \longmapsto e^\alpha(X^\Lambda, F_\Lambda) \quad , \quad \bar{\partial}_i \alpha = 0 \tag{4}$$

the special coordinates t^i and the prepotential $f(t)$ are invariant.

Property (4) is crucial in defining real coordinates. In fact, while it makes sense to define the real part of the holomorphic special coordinates t^i, it would not make sense to define a real part of X^Λ since this operation would be incompatible with the Hodge structure of the manifold. Note that this is also the main difference between the local and the rigid cases [18], [19], [14], [24], since in the latter the Hodge structure is absent and this obstruction does not arise. We also note that in local special coordinates the $\mathbf{Sp}(2n + 2, \mathbb{R})$ covariance is lost (by the Kähler gauge-fixing $X^0 = 1$) but still a $\mathbf{Sp}(2n, \mathbb{R})$ structure is present with respect to the reduced $2n$-dimensional holomorphic section

$$V_R = \left(t^i, f_i = \frac{\partial f}{\partial t^i}\right) \tag{5}$$

In particular, in terms of the matrix $f_{ij} = \frac{\partial^2 f}{\partial t^i \partial t^j}$, we can construct a real symmetric $2n \times 2n$ symplectic matrix $M(f_{ij})$ as follows [26], [12]:

$$M(f_{ij}) = \begin{pmatrix} \Im f_{ij} + \Re f_{ik} \Im(f^{-1})^{kl} \Re f_{lj} & -\Re f_{ik} \Im(f^{-1})^{kj} \\ -\Im(f^{-1})^{ik} \Re f_{kj} & \Im(f^{-1})^{ij} \end{pmatrix} \tag{6}$$

with the obvious property $M\Omega M = \Omega$.

We note at this point that the $M(f_{ij})$ matrix considered here is not the $M(\mathcal{F})$ matrix considered in [26], [12]. Indeed while $M(\mathcal{F}) \in \mathbf{Sp}(2n+2, \mathbb{R})$ is a $(2n+2) \times (2n+2)$ symplectic matrix, the matrix considered here is $2n \times 2n$ and it is $\mathbf{Sp}(2n, \mathbb{R})$ symplectic. However, it is this last matrix which plays a role in our considerations.

The Kähler potential of special geometry in these coordinates is

$$\begin{aligned} K &= -logY \tag{7} \\ Y &= i\left(2f - 2\bar{f} - (t^i - \bar{t}^i)(f_i + \bar{f}_i)\right) \tag{8} \end{aligned}$$

from which the following expression for the metric holds:

$$g_{i\bar{j}} = \partial_i \bar{\partial}_j K = -\frac{1}{Y}Y_{i\bar{j}} + \frac{1}{Y^2}Y_i Y_{\bar{j}} \tag{9}$$

By explicit computations the two terms in (9) give:

$$K_{i\bar{j}} = -\frac{i}{Y}(f_{ij} - \bar{f}_{ij}) + \frac{1}{Y^2}\left((f_i - \bar{f}_i + (\bar{t}^k - t^k)f_{ki}) \times (\bar{f}_j - f_j + (t^k - \bar{t}^k)\bar{f}_{kj})\right) \tag{10}$$

We now go to Darboux special coordinates exactly as we did in the rigid case [24] i.e. we define

$$t^i = p^i + i\phi^i \quad , \quad f_i = q_i + i\psi_i \tag{11}$$

and the Legendre transform $S(p, q)$ of the imaginary part of the prepotential

$$L = \Im f \tag{12}$$

Then the (q_i, ϕ^i) and (p^i, ψ_i) real sections are pairs of conjugate variables for L

$$q_i = \frac{\partial L}{\partial \phi^i} \quad , \quad \psi_i = \frac{\partial L}{\partial p^i} \tag{13}$$

and in terms of S given by

$$S(p, q) = q_i \cdot \phi^i(p, q) - L(p, \phi(p, q)) \tag{14}$$

we have the following set of equations

$$\phi^i = \frac{\partial S}{\partial q_i} \quad , \quad \psi_i = -\frac{\partial S}{\partial p^i} \tag{15}$$

4

By comparing (14) and (8) we realize that [15], [17], [27], [16]

$$Y = 4S \tag{16}$$

i.e.

$$S = \frac{1}{4}e^{-K} \tag{17}$$

Eq. (17) is the main difference between rigid and local special geometry. We just recall that in the former the (square of the) distance ds^2 is given by [18], [19], [24], [25]

$$K_{ij}dz^i \otimes d\bar{z}^j = -2H_{IJ}dP^I \otimes dP^J \tag{18}$$

where $H_{IJ} = \frac{\partial^2 S}{\partial P^I \partial P^J}$ is the Hessian of the S functional. In the next section we will see how the rigid formula (18) is modified in the local case.

The change of variables from the holomorphic sections (t^i, f_i) to the Darboux variables (p^i, q_i) and (ϕ^i, ψ_i) is exactly as in the rigid case it is still true [24] that the H matrix

$$H = \begin{pmatrix} \frac{\partial^2 S}{\partial p^i \partial p^j} & \frac{\partial^2 S}{\partial p^i \partial q_j} \\ \frac{\partial^2 S}{\partial q_i \partial p^j} & \frac{\partial^2 S}{\partial q_i \partial q_j} \end{pmatrix} = \begin{pmatrix} S_{ij} & S_i^j \\ S_j^i & S^{ij} \end{pmatrix} \tag{19}$$

is equal to the (negative of the) M matrix defined in (6), then it follows that $H\Omega H = \Omega$ i.e.

$$\begin{aligned} S_i^k S_{kj} - S_j^k S_{ki} &= 0 \\ S^{ik} S_k^j - S^{jk} S_k^i &= 0 \\ S_{ik} S^{kj} - S_i^k S_k^j &= \delta_i^j \end{aligned} \tag{20}$$

By comparing the M^{ij} and M_j^i entries of M with H it also follows that

$$f_{ij} = -(S_i^k + i\delta_i^k)(S^{-1})_{kj} \tag{21}$$

where $(S^{-1})_{kj} = (S^{kj})^{-1}$ is the inverse of S^{kj} defined by (19).

We can now write Eq. (10) in Darboux special coordinates by means of Eqs. (11), (13), and (21) and we finally obtain

$$K_{i\bar{j}} = -\frac{1}{2S}(S^{ij})^{-1} + \frac{1}{4S^2}\left(\frac{\partial S}{\partial p^i}\frac{\partial S}{\partial p^j} + \frac{\partial S}{\partial q_k}\frac{\partial S}{\partial q_l}f_{ki}\bar{f}_{lj} + \frac{\partial S}{\partial q_k}\frac{\partial S}{\partial p^j}f_{ki} + \frac{\partial S}{\partial q_k}\frac{\partial S}{\partial p^i}\bar{f}_{kj} \right) \tag{22}$$

where f_{ij} is given by (21).

We just note that while in the rigid case $K_{i\bar{j}} = -2(S^{ij})^{-1}$, in the local case (22) also depends on S and the canonical momenta $S_I = \frac{\partial S}{\partial P^I}$. The second term in (22) is the main difference between the rigid and local case.

5

3 The real symplectic metric

In order to compute the real metric we have to transform the "distance" from holomorphic to Darboux special coordinates

$$ds^2 = K_{i\bar{j}}dt^i \otimes d\bar{t}^j = g_{IJ}dP^I \otimes dP^J \tag{23}$$

The result is obtained by computing the differentials $dt^i \otimes dt^j$ in Darboux coordinates and then comparing the two expressions in Eq. (23).
We just mention that the computation of $dt^i \otimes d\bar{t}^j$ is exactly the same as in the rigid case [24] since the Legendre transform and the conjugate variables are the same. Indeed only Eqs. (19), (20), (21) which are the same as in rigid special geometry really matter.

The change of variables in the differentials was given in Eq. (50) of reference [24] and it is reproduced here for the benefit of the reader.

$$
\begin{aligned}
dt^i \quad \otimes \quad d\bar{t}^j &= \left(\delta_k^i \delta_l^j + \frac{\partial^2 S}{\partial q_i \partial p^k} \frac{\partial^2 S}{\partial q_j \partial p^l} + i \left(\frac{\partial^2 S}{\partial q_i \partial p^k} \delta_l^j - \frac{\partial^2 S}{\partial q_j \partial p^l} \delta_k^i \right) \right) dp^k \otimes dp^l \\
&+ \left(\frac{\partial^2 S}{\partial q_i \partial q_l} \frac{\partial^2 S}{\partial q_j \partial p_k} + \frac{\partial^2 S}{\partial q_i \partial p_k} \frac{\partial^2 S}{\partial q_j \partial q_l} + i \left(\frac{\partial^2 S}{\partial q_i \partial q_l} \delta_k^j - \frac{\partial^2 S}{\partial q_j \partial q_l} \delta_k^i \right) \right) dp^k \otimes dq_l \\
&+ \left(\frac{\partial^2 S}{\partial q_i \partial q_k} \frac{\partial^2 S}{\partial q_j \partial q_l} \right) dq_k \otimes dq_l
\end{aligned}
\tag{24}
$$

By explicit multiplication of (22) with (24) we finally obtain, from the first term in (22)

$$-\frac{1}{2S}(S^{ij})^{-1}dt^i \otimes d\bar{t}^j = -\frac{1}{2S}H_{IJ}dP^I \otimes dP^J \tag{25}$$

For the second term of (22) (by multiplying by $dt^i \otimes d\bar{t}^j$) we find

$$-\frac{1}{8S^2} \left(S^K H_{KI} S_J - S^K H_{KJ} S_I \right) \left(dP_L H^{LI} dP^J - dP_L H^{LJ} dP^I \right) \tag{26}$$

where

$$S^K = \Omega^{KI} S_I \quad , \quad H^{LJ} = (H_{LJ})^{-1} \quad , \quad dP_L = \Omega_{LI} dP^I$$

and

$$\Omega^{KI} = (\Omega_{KI})^{-1} = -\Omega_{KI}$$

By multiplying the four terms in (26) we finally obtain:

$$\frac{1}{4S^2} \left(S_I S_J + (H\Omega S)_I (H\Omega S)_J \right) \tag{27}$$

where

$$(H\Omega S)_I = H_{IK} \Omega^{KL} S_L$$

The final expression for the real symmetric metric in Darboux coordinates, for local special geometry, is therefore given by

$$g_{IJ}(P) = -\frac{1}{2S}H_{IJ} + \frac{1}{4S^2} \left(S_I S_J + (H\Omega S)_I (H\Omega S)_J \right) \tag{28}$$

We note at this point that a major simplification occurs for $I = 1, 2$ which corresponds to one-dimensional complex special geometry. Indeed in this case

$$S^K H_{KI} S_J - S^K H_{KJ} S_I = -\Omega_{IJ} S_K (H^{-1})^{KL} S_L \tag{29}$$

$$dP_L H^{LI} dP^J - dP_L H^{LJ} dP^I = -\Omega^{IJ} dP^L H_{LK} dP^K \tag{30}$$

and then (26) becomes

$$\frac{1}{4S^2} S_K (H^{-1})^{KL} S_L H_{IJ} dP^I \otimes dP^J \tag{31}$$

In this case, the metric therefore is

$$g_{IJ} = \left(-\frac{1}{2S} + \frac{1}{4S^2}(SH^{-1}S) \right) H_{IJ} \tag{32}$$

We finally note that the metric given by (28) has a manifest $\mathbf{Sp}(2n, \mathbb{R})$ covariant structure. The simplification occurring in (29) is due to the fact that for $\mathbf{Sp}(2, \mathbb{R})$ the antisymmetric representation is the identity while the traceless part is non-vanishing for $n > 1$; and this is the reason why (28) and (32) are equivalent only for $n = 1$.

It is obvious that in special coordinates the $\mathbf{Sp}(2n+2, \mathbb{R})$ structure of the geometry is lost and only a $\mathbf{Sp}(2n, \mathbb{R})$ structure is manifest. Nevertheless we have shown that this is what is needed to obtain a general expression for the metric in terms of the Darboux data of local special geometry. The $\mathbf{Sp}(2n+2, \mathbb{R})$ covariance will be considered elsewhere.

4   Some examples

In this section we compute the real metric in some examples of special geometry. The two examples in question correspond to a cubic and quadratic prepotential, for one complex scalar:

$$f(t) = \frac{1}{3}t^3 \tag{33}$$

$$f(t) = \frac{i}{4}(t^2 - 1) \tag{34}$$

Let us first consider the cubic case. The S functional is

$$4S = Y = -\frac{8}{3}(\Im t)^3 \tag{35}$$

where $S = -\frac{2}{3}(\Im t)^3$ so that $\Im t < 0$. Note that in this case $\Im f_{tt} = 2\Im t < 0$ and therefore the Hessian will be positive definite. Going to Darboux coordinates we find

$$q = p^2 - \phi^2 \longrightarrow \phi = \pm(p^2 - q)^{\frac{1}{2}} \tag{36}$$

so we have $p^2 > q$ and also the negative root must be chosen such that S is positive. From (36) and (35) we obtain

$$S = \frac{2}{3}(p^2 - q)^{\frac{3}{2}} \tag{37}$$

7

The Hessian matrix is given by

$$H = (p^2 - q)^{-\frac{1}{2}} \begin{pmatrix} 2(2p^2 - q) & -p \\ -p & \frac{1}{2} \end{pmatrix} \tag{38}$$

Since $Det(H) = 1$ and also $(p^2 - q)^{\frac{1}{2}} Tr(H) = 2(2p^2 - q) + \frac{1}{2} > 0$, H is indeed positive definite. We can now compute the metric by using (32). First, by computing the second term, we obtain

$$SH^{-1}S = 2(p^2 - q)^{\frac{3}{2}} \tag{39}$$

and then

$$-\frac{1}{2S} + \frac{SH^{-1}S}{4S^2} = \left(-\frac{3}{4} + \frac{9}{8}\right)(p^2 - q)^{-\frac{3}{2}} = \frac{3}{8}(p^2 - q)^{-\frac{3}{2}} \tag{40}$$

so that the metric is

$$g_{IJ}(p, q) = \frac{3}{8}(p^2 - q)^{-2} \begin{pmatrix} 2(2p^2 - q) & -p \\ -p & \frac{1}{2} \end{pmatrix} \tag{41}$$

We now turn to the second example (34) where now

$$Y = 4S = 1 - t\bar{t} = 1 - p^2 - \phi^2 \tag{42}$$

therefore we must take $p^2 + \phi^2 < 1$. Since

$$\Im f = \frac{1}{4}(p^2 - \phi^2 - 1) = L$$

we have

$$q = \frac{\partial L}{\partial \phi} = -\frac{1}{2}\phi \qquad \longrightarrow \qquad \phi = -2q$$

It follows that

$$Y = 4S = 1 - p^2 - 4q^2 \quad , \quad S = \frac{1}{4} - \frac{p^2}{4} - q^2 \tag{43}$$

The Hessian matrix is

$$H = \begin{pmatrix} -\frac{1}{2} & 0 \\ 0 & -2 \end{pmatrix} \tag{44}$$

and it is negative definite. The conjugate momenta are

$$\frac{\partial S}{\partial p} = -\frac{1}{2}p \quad , \quad \frac{\partial S}{\partial q} = -2q$$

We then have

$$SH^{-1}S = -\frac{1}{2}(p^2 + 4q^2)$$

and the conformal factor becomes

$$-2\frac{1}{(1 - p^2 - 4q^2)^2}$$

The local metric is then finally given by

$$g_{IJ}(p, q) = \frac{1}{(1 - p^2 - 4q^2)^2} \begin{pmatrix} 1 & 0 \\ 0 & 4 \end{pmatrix} \tag{45}$$

8

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